Melting properties of $\mathrm{He^3}$ and $\mathrm{He^4}$



FIG. 10. $(\partial P/\partial T)_V$ and $(C_P - C_V)$ for fluid He³ along the melting curve.

lowest accessible mean temperature for the present α_f measurements. It is seen that the curve of Fig. 5 intersects the melting curve at 47 kg/cm² in good agreement with the extrapolations made in Figs. 4 and 10. Temperatures where $\alpha_f = 0$, derived from pressure-volume-temperature data by Brewer and Daunt (28) and Sherman and Edeskuty (29), are in general agreement with the measurements of Fig. 5.

The slopes, $(\partial \alpha_f / \partial T)_P$ and $(\partial \beta_f / \partial P)_T$, decrease with increasing melting pressure as shown in Figs. 6 and 7, respectively. From the thermodynamic formulas,

$$(\partial C_P / \partial P)_T = -T(\partial^2 V / \partial T^2)_P = -TV[\alpha^2 + (\partial \alpha / \partial T)_P]$$
(7)

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and the data given above for fluid He³, it can be seen that C_P decreases with increasing pressure in the vicinity of the melting curve. Gutsche (30) and Jones and Walker (31) reported a similar variation for H₂ and A, respectively.

Equation (5) combined with the thermodynamic relation,

$$\frac{dP_m}{dT_m} = \left[\left(\frac{\partial P}{\partial V} \right)_r \right]_m \frac{dV_m}{dT_m} + \left[\left(\frac{\partial P}{\partial T} \right)_v \right]_m, \tag{8}$$

leads to

$$\beta_f = \alpha_f \frac{dT_m}{dP_m} - \frac{1}{V_f} \frac{dV_f}{dP_m}.$$
(9)

For He³ at low pressures, the values of β_f calculated from Eq. (9) compare reasonably well with those measured directly (Fig. 4), deviating by +27 percent at $P_m = 50 \text{ kg/cm}^2$ and by -2 percent at $P_m = 225 \text{ kg/cm}^2$. At 3555 kg/cm², the calculated β_f is 7.4 $\times 10^{-5} \text{ cm}^2/\text{kg}$.

The theory of melting for metals that was advanced by Bonfiglioli *et al.* (32) predicts that $\alpha_s T_m$ is constant for a given crystal type. Unfortunately, available data are restricted to melting pressures of ~1 atmos but for face-centered-cubic, body-centered-cubic, and hexagonal-closest-packed metals of widely varying melting temperature, $\alpha_s T_m$ appears to be ~0.06. Above the anomalous region where α_f shows a maximum, the present results for fluid He³ (and He⁴) indicate that values of $\alpha_f T_m$ rise rapidly with P_m then approach constancy around 0.05–0.06 at high melting pressures. The empirical deduction from the present work that $\alpha_s = 0.75\alpha_f$ indicates that the expansion of the solid along the melting curve follows closely that of the fluid and implies a constant value of 0.04–0.05 for $\alpha_s T_m$ at high pressures. It is interesting to compare the ratio $\alpha_s/\alpha_f = 0.75$ for He³ and He⁴ with the ratios 0.70 and 0.77 for Na and K, respectively, measured by Bridgman (33) at $P_m = 1$ kg/cm².

Values of V_s can be calculated from the present measurements of V_f and ΔV_m . For He³ the ratio V_f/V_s was found to be constant and equal to 1.044 with a maximum deviation of only 0.4 percent over the full pressure range studied.⁴ Therefore $(1/V_f)(dV_f/dP_m) = (1/V_s)(dV_s/dP_m)$ which, with Eq. (8) and the ratio $\alpha_s/\alpha_f = 0.75$, leads to the following equations for He³:

$$\beta_s = 0.75\alpha_f \frac{dT_m}{dP_m} - \frac{1}{V_f} \frac{dV_f}{dP_m}, \qquad (10)$$

and

$$\Delta\beta = \beta_f - \beta_s = 0.25\alpha_f (dT_m/dP_m). \tag{11}$$

⁴ For He⁴ the ratio of V_f/V_s varied monotonically from 1.066 at $P_m = 35 \text{ kg/cm}^2$ to 1.044 at $P_m = 3555 \text{ kg/cm}^2$.